- **Projectile Motion**

Inquiry question: How can models that are used to explain projectile motion be used to analyse and make predictions?

Analysing Projectile Motion

Objects that are close to the Earth's surface accelerate toward the surface of the Earth as a result of their weight force. The weight force is due to the Earth's gravitational field which is considered a constant for objects that are near the Earth's surface ($g = 9.8m/s^2$).

As g is constant for all objects, all objects must accelerate toward the Earth at the same rate. This is not apparent in real life (consider dropping a feather and a rock at the same time – a rock will hit the ground first). This is because there is a net force acting on any object falling toward the Earth. This net force is a result of the weight force of the object acting down and a resistive force acting upward. The resistant force that opposes the motion of a falling object is known as air resistance or drag (F_{drag}). The drag force increases as the objects speed increases.

The drag that an object experiences as it accelerates toward Earth is proportional to:

- The area of the object perpendicular to the direction of motion
- The density of the air through which the object moves
- The objects speed

The net force acting on an object falling toward the Earth under the influence of gravity is:

$$F_{net} = W + F_{drag}$$

Terminal velocity

An object that accelerates toward Earth experiences a constant weight force, *W*. The drag force (F_{drag}) that the object experiences, increases to a point where it is equal but opposite in direction to the weight force. At this point the net force (F_{net}) acting on the object is zero and it will no longer accelerate and its velocity remains constant. This velocity is called the **terminal velocity.**

Analysis of Projectile Motion

Characteristics of projectile motion:

- An object is given an initial velocity
- The object continues to move due to its own inertia and its weight force
- The projectile follows a parabolic path (ignoring air resistance)

When we analyse projectile motion problems, we make two assumptions:

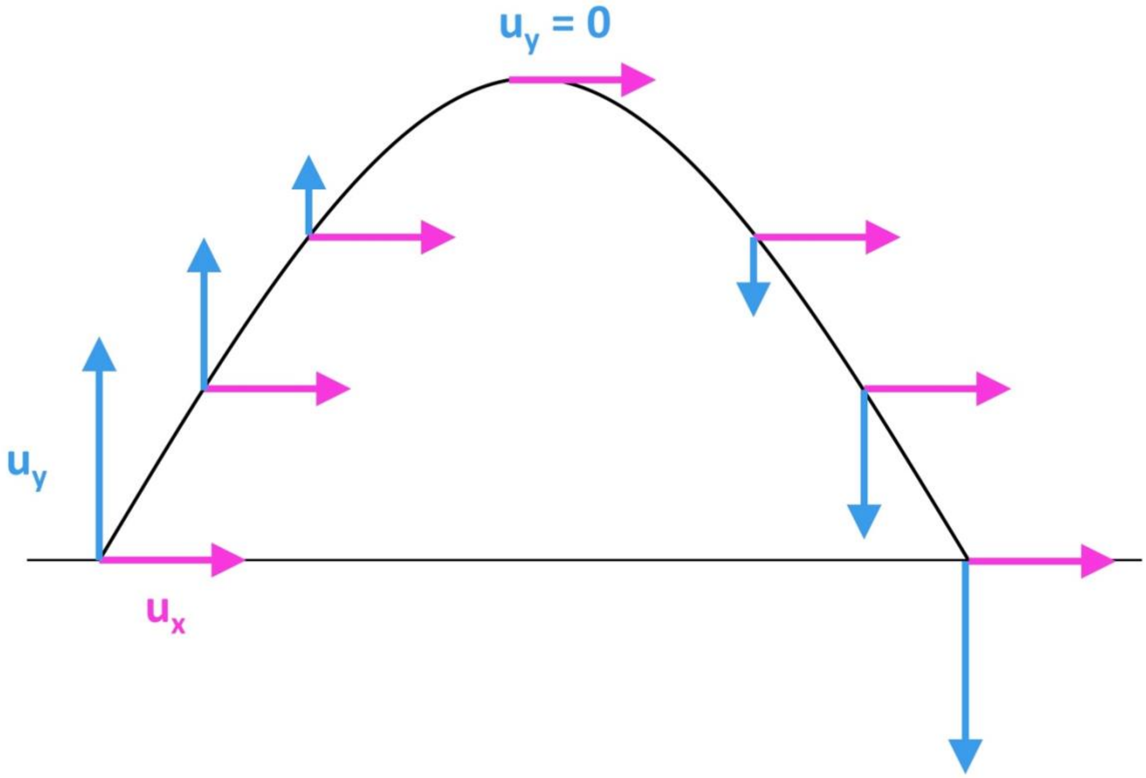
- All objects experience a constant vertical acceleration due to gravity ($g = 9.8m/s^2$)
- All objects experience zero air resistance

Modelling Projectile Motion

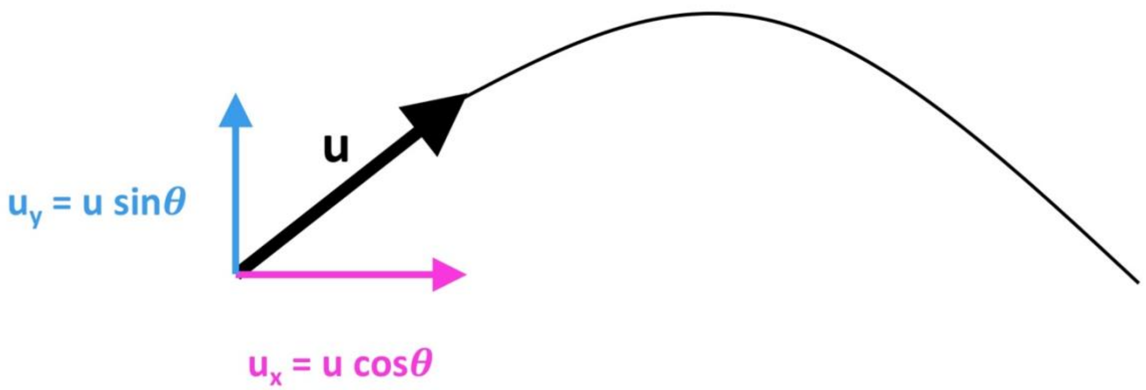
The following assumptions are made for all projectiles analysed in this course:

- For projectiles launched horizontally the initial vertical velocity is 0
- For projectiles launched vertically the initial and final horizontal velocity is 0
- The horizontal component of the projectile's velocity is constant
- The vertical component of the projectiles velocity constantly changes

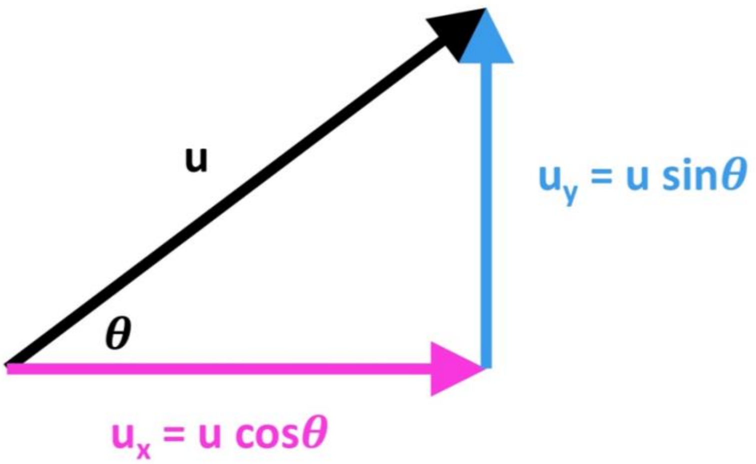
The diagram below illustrates how the horizontal and vertical velocity vectors vary over time. Note that the horizontal vector is constant, and the vertical vector is zero at the top of the trajectory and it changes direction and magnitude:



When modelling projectile motion problems, the horizontal and vertical vector components are analysed separately. The diagram below illustrates how the initial velocity, *u*, is resolved into its horizontal and vertical components:



This is usually simplified to the diagram below for analysis:



Projectile Motion Equations

The following equations are applied to projectile motion problems:

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

Where:

u = initial velocity in m/s

v = final velocity in m/s

t = time in sec

s = displacement in m

a = acceleration due to gravity ($g = 9.8m/s^2$)

Equations and vector analysis
As projectile motion problems are analysed in their horizontal and vertical vector components, the equations need to be written with subscripts to reflect this analysis – for example:

$$v_y = u_y + a_y t$$

The importance of direction

Projectile motion deals with many variables which are vectors. This means that direction is a very important consideration for the analysis of projectile motion problems.

- **Horizontal:** As projectiles will only ever move in one direction horizontally, we naturally make this direction positive.
- **Vertically:** As projectiles can move in both directions vertically, a direction (up or down) must be noted as positive. All problems analysed here will consider down as positive*. A key result of this is that the acceleration due to gravity will always be positive ($g = 9.8m/s^2$).

*It does not matter which direction you choose to be positive; both will calculate the same answer if direction is consistent throughout the working.

The following are common values that may need to be derived in many projectile motion problems:

- **Maximum height reached** (Δy)
- **Time of flight** (t)
- **Range** (Δx)

These variables are often the link to solving more difficult problems consisting of several parts.

Maximum height reached (Δy)

The maximum height reached considers the vertical part of the projectiles motion. A key point here is that at the maximum height the vertical velocity will be 0.

Using the equation: $v^2 = u^2 + 2as$ and writing this with vertical subscripts:

$$v_y^2 = u_y^2 + 2a_y\Delta y$$

Considering the following:
- $v_y = 0$ (vertical velocity is 0 at maximum height)
- $u_y = u\sin\theta$ (vertical vector of initial velocity, u)
- $a_y = g = 9.8m/s^2$

We derive the following equation for maximum height:

$$0 = u_y^2 + 2a_y\Delta y$$

$$0 = (u\sin\theta)^2 + 2g_y\Delta y$$

$$\Delta y = \frac{-(u\sin\theta)^2}{2g}$$

Time of flight (t)

For a projectile that starts and finishes its trajectory at the same height the total flight time will be 2× the time the projectile takes to reach its maximum height: $t_{total} = 2t_{max\ height}$
Using the equation: and writing this with vertical subscripts:

$$v_y = u_y + a_y t$$

Considering the following:

- $v_y = 0$ (vertical velocity is 0 at maximum height)
- $u_y = u\sin\theta$ (vertical vector of initial velocity, u)
- $a_y = g = 9.8m/s^2$

We derive the following equation for the time to reach maximum height:

$$0 = u\sin\theta + gt$$

$$t = \frac{-u\sin\theta}{g}$$

Considering: $t_{total} = 2t_{max\ height}$

$$t_{total} = \frac{-2u\sin\theta}{g}$$

Range $(\Delta x$)

The range of a projectile considers the horizontal part of the projectiles motion. A key point here is that the projectile has a constant horizontal velocity

Using the equation: $s = ut + \frac{1}{2}at^2$ and writing this with horizontal subscripts:

$$\Delta x = u_x t + \frac{1}{2}a_x t^2$$

Considering the following:

- $u_x = u\cos\theta$ (horizontal vector of initial velocity, u)
- $a_x = 0$ (horizontal velocity is constant)

We derive the following equation for the range:

$$\Delta x = u_x t + \frac{1}{2} \times 0 \times t^2$$

$$\Delta x = u_x t$$

Projectile Motion Problems

There is a large number of possible Projectile motion problems you may be required to solve. They can vary in the type of information you are given, the questions that you are asked and also the flight of the projectile. The projectile problem may involve:

- Projectiles that launch horizontally or vertically
- Projectiles that launch and land at the same height
- Projectiles that launch and land at different heights
- Projectiles that hit a boundary (cliff or building)

Projectile motion problems will usually require you to answer multiple parts or solve a problem using several steps. In any case, developing a method or technique you can use for every problem allows you to become more confident in completing problems.

General technique for analysing projectile motion problems:

1. Read through the question and draw a diagram that reflects the motion of the projectile (some questions may give you a diagram)
2. Fill in any information on your diagram that you can determine from the question (initial velocity, launch angle, height of cliff, etc)
3. Denote a direction as positive
4. Write a list of the variables you have been given in the question
5. For each part of the question, write the variable you are trying to determine
6. Determine the equation you will use to solve the problem