## HSC Physics - Module 5: Advanced Mechanics - Motion in Gravitational Fields Short Answer Question Solutions 3

21. (a)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Substitutes into correct equation to calculate the orbital speed | 2 |
| $\bullet$ Attempts to calculate the orbital speed using the correct formula OR | 1 |
| $\bullet$ Correctly converst the time into seconds |  |

Sample answer:
$t=7$ hours 39 minutes $=(7 \times 3600)+(39 \times 60)=27540 \mathrm{~s}$
$r=9.5 \times 10^{6} \mathrm{~m}$
$v=\frac{2 \pi r}{T}=\frac{2 \pi\left(9.5 \times 10^{6}\right)}{27540}=2167.402 \ldots=2.2 \times 10^{3} \mathrm{~ms}^{-1}$
(b)

| Criteria | Marks |
| :---: | :---: |
| Response includes reference to the following points: <br> - Relates atmospheric thickness at lower altitudes to greater air resistance on satellites in LEO <br> - Relates air resistance on satellite to a decrease in its orbital speed (or heat and kinetic energy) <br> - Relates a decrease in orbital speed (or kinetic energy) to a decrease in altitude OR <br> - Relates cause and effect to two different issues such as air resistance and tracking stations | 3 |
| Response includes most of the above points | 2 |
| Response includes one of the above points | 1 |

Sample answer:

Satellites in low Earth orbits are affected by air resistance because the atmosphere is thicker near the Earth's surface due to gravity. The air resistance causes the satellites orbital speed to decrease. This decrease in speed causes the satellite to decrease in altitude. (As the satellite drops in altitude the atmosphere is even thicker so provides greater air resistance on the satellite slowing it down even further. The heating effects associated with such air resistance can cause a satellite to eventually burn up.
(c)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Correctly calculates the orbital radius | 2 |
| $\bullet$ Attempts to calculate the orbital radius | 1 |

Sample answer:
$\mathrm{T}=24$ hours $=24 \times 3600=86400 \mathrm{~s}$
$M=6.0 \times 10^{24} \mathrm{~kg}$
$\frac{r^{3}}{T^{2}}=\frac{G M}{4 \pi^{2}}$
$\frac{r^{3}}{86400^{2}}=\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{4 \pi^{2}}$
$r=42297523.87 \ldots$
$=4.2 \times 10^{7} \mathrm{~m}$
22. (a)

| Criteria | Marks |
| :--- | :--- | :--- |
| - $\quad$Identifies that the astronauts in the ISS are still within the gravitational field of <br> the Earth, hence as they have mass they still have a weight. <br> - Identifies that the astronauts will feel the sensation of being 'weightless' as they <br> are in free fall around the Earth <br> Response includes a correct assessment of the validity, justified based on the <br> reasoning presented | 3 |
| -Identifies that the astronauts in the ISS are still within the gravitational field of <br> the Earth, hence as they have mass they still have a weight OR | 2 |
| -Identifies that the astronauts will feel the sensation of being 'weightless' as they <br> are in free fall around the Earth |  |
| - Response includes a correct assessment of the validity, justified based on the |  |
| reasoning presented |  |$\quad$| Identifies that the astronauts in the ISS are still within the gravitational field of |
| :--- |
| the Earth, hence as they have mass they still have a weight OR |
| - Identifies that the astronauts will feel the sensation of being 'weightless' as they |
| are in free fall around the Earth |

(b)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Correct answer with correct formula substitution and units | 3 |
| $\bullet$ Correct substation with convert km $\rightarrow \mathrm{m}$ | 2 |
| $\bullet$ Answer provided with one error OR | 1 |
| • Incorrect substitution |  |

$F=G \frac{m_{1} m_{2}}{d^{2}}$
$\frac{F}{m_{1}}=a=G \frac{m_{\text {planet }}}{d^{2}}$
$=6.67 \times 10^{-11} \times \frac{2.50 \times 10^{24}}{\left(4.00 \times 10^{8}\right)^{2}}$
$=1.04 \times 10^{-3} \mathrm{~ms}^{-1}$
23. (a)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Correct answer calculated. (units not graded) | 2 |
| $\bullet$ Correct substitution or error made with metric conversion | 1 |

$T=2 \times 60 \times 60=7200 \mathrm{~s}$
$\frac{r^{3}}{T^{2}}=\frac{G M}{4 \pi^{2}}$
$r=\sqrt[3]{\frac{G M}{4 \pi^{2}} \times T^{2}}$
$=\sqrt[3]{\frac{\left(6.67 \times 10^{-11}\right)\left(6.0 \times 10^{24}\right)}{4 \pi^{2}} \times 7200^{2}}$
$=8.1 \times 10^{6} \mathrm{~m}$

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(b)
$F=\frac{G m_{1} m_{2}}{d^{2}}$
$=6.67 \times 10^{-11} \times \frac{\left(4.00 \times 10^{2}\right) \times\left(6.0 \times 10^{24}\right)}{\left(8.07 \times 10^{6}\right)^{2}}$
$=2.46 \times 10^{3} \mathrm{~N}$

## Note: allow carry-over error from part (a)

(1 mark for correct formula selection and substitution, and one mark for correct answer. Direction not necessary to gain full marks)
(c)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ All appropriate reasons given | 2 |
| $\bullet$ One appropriate reason given | 1 |

Sample answer:

Satellite's orbit is circular and the net force (gravitational force) is always perpendicular to the direction of motion and directed to the centre of the Earth, i.e. the centre of the orbital path. (ie, the gravitational force provides the necessary centripetal force, directed towards the centre of the Earth)
(d)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Describing that an empty stage can be abandoned and less mass to carry | 1 |

Sample answer:

Once the fuel inside a rocket motor is exhausted, the motor is 'dead-weight', and a great deal of energy would be wasted to carry it further, so the 'stage' is allowed to drop away.
24.

| Marking Criteria | Marks |
| :---: | :--- |
| $\bullet$ Both changes described | 2 |
| $\bullet$ One change described or both changes (period and radius) identified | 1 |

To achieve a shorter period (travel faster) to keep up with Earth's rotation, the orbital radius of geostationary satellites would be reduced.
25. (a)

| Marking Criteria | Marks |
| :---: | :--- |
| $\bullet$ Gives the correct answer | 2 |
| $\bullet$ Answer involves one error in working | 1 |

Radius of orbit $=3700 \mathrm{~km}=3.7 \times 10^{6} \mathrm{~m}$
$\frac{r^{3}}{T^{2}}=\frac{G M}{4 \pi^{2}}$
$\frac{\left(3.7 \times 10^{6}\right)^{3}}{T^{3}}=\frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{4 \pi^{2}}$
From whence we derive $T=6833.6 \mathrm{~s}$
(b)

| Marking Criteria | Marks |
| :--- | :--- |
| $\bullet$ Gives the correct answer | 3 |
| $\bullet$ Error in working (especially wrong mass or radius used) or left out direction | 2 |
| $\bullet$ Makes two errors in working | 1 |

Using $C=2 \times r=2 \times\left(3.7 \times 10^{6}\right)=2.32 \times 10^{7} \mathrm{~m}$ (the distance travelled by the MRO). Now to determine the velocity, use this figure and the answer from part (a):
$v=\frac{d}{t}=\frac{2.32 \times 10^{7}}{6833.6}=3395 \mathrm{~ms}^{-1}$
Now find the centripetal force using
$F=\frac{m v^{2}}{r}$
$=\frac{1030 \times 3395^{2}}{3.7 \times 10^{6}}=3209 \mathrm{~N}$
Note there are Alternative approaches:

Alternative approach \#1:
$v=\sqrt{\frac{G M}{r}}$
$V=3401.96 \mathrm{~ms}^{-1}$
Then use $F_{c}=\frac{m v^{2}}{r}$
$\mathrm{F}=3221.8 \mathrm{~N}$

Alternative approach \#2:
Equate gravitational force with centripetal force
$F_{c}=F_{G}$
$\frac{m v^{2}}{r}=\frac{G m_{1} m_{2}}{d^{2}}$
This will give $\mathrm{F}=3221.7 \mathrm{~N}$

| Marking Criteria | Marks |
| :---: | :---: |
| - sound understanding of relevant factors and relationships evident <br> - appropriate energy formulae referred to <br> - Logical presentation | 3 |
| - basic understanding of the relevant factors and relationships evident, <br> - some appropriate formulae identified | 2 |
| - Some understanding of a relevant concept evident (such as quoting the escape velocity formula) | 1 |

26. 

Sample answer: For a spacecraft to escape Earth's gravitational field it must either: possess sufficient kinetic energy, $E_{k}=1 / 2 m v^{2}$, so that it can gain gravitational potential energy, $\mathrm{sefi} \mathrm{E}_{\mathrm{p}}=-\mathrm{Gm} m_{1} / \mathrm{r}$ of at
least zero ; or be able to propel itself with sufficient fuel so that it can "climb up" to a potential energy of zero. For a projectile at launch, EK equal to or greater than $\mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}$.

Alternative approach: Discuss the factors involved in achieving escape velocity as outlined in the escape velocity formula. BUT...candidates still need to discuss energy factors, as this is explicitly asked for in the question.
27. (a)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Correct answer | 1 |

Gravity is the force.
(b)

| Criteria | Marks |
| :---: | :--- |
| $\bullet$ Complete solution using both F(centripetal) and F(Gravity) | 2 |
| $\bullet$ Partial solution using either F(centripetal) and F(gravity) | 1 |

F (centripetal) and F (gravity) becomes $m\left(\frac{v^{2}}{r}\right)=G M\left(\frac{m^{2}}{r}\right)$
Simplifies to $v^{2}=\frac{G M}{r}$
Giving $v=\left(\frac{G M}{r}\right)^{\frac{1}{2}}$
28.

| Criteria | Marks |
| :--- | :--- |
| - Correct formula, correct substitution, correct answer | 3 |
| - Correct formula, incorrect substitution or incorrect rounding | 2 |
| - Correct formula only | 1 |

Sample answer:
$\frac{m_{1} v^{2}}{r}=\frac{G m_{1} m_{E}}{r^{2}}$
$v=\sqrt{\frac{G m_{E}}{r+h}}$
$=\sqrt{\frac{6.67 \times 10^{-11} .6 \times 10^{24}}{6.37 \times 10^{6}+350 \times 10^{3}}}$
$=7717.1 \mathrm{~ms}^{-1}$
$=7720 \mathrm{~ms}^{-1}(3 \mathrm{sf})$
$=7.72 \mathrm{~km} / \mathrm{s}$ (3sf)
29.

| Criteria | Marks |
| :---: | :--- |
| - Correct formula, correct substitution, correct calculation | 2 |
| - Correct formula, incorrect change of subject of formula, correct calculation | 1 |

Sample answer:
$\frac{r_{1}^{3}}{T_{1}^{2}}=\frac{r_{2}^{3}}{T_{2}^{2}}$
$\frac{(4.2)^{2}}{(1.8)^{2}}=\frac{r_{2}^{3}}{(16.7)^{2}}$
$r_{2}^{3}=\frac{4.2^{3}}{1.8^{2}} \times 16.7^{2}$
$r_{2}=18.5$ units = distance to Calisto (from centre of Jupiter)
30.
(a) $\frac{R^{3}}{T^{2}}=\frac{G M}{4 \pi^{2}}$
$T=\sqrt{R^{3} \times \frac{4 \pi^{2}}{G M}} R=3.39 \times 10^{6}+5 \times 10^{5}=38.9 \times 10^{5}$
$=\sqrt{\frac{\left(3.89 \times 10^{6}\right)^{3} \times 4 \pi^{2}}{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}}$
$=7366.714 \mathrm{~s}$
$=2.04$ hours
(b) $g=\frac{G M}{r^{2}}$
$=\frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{38.9 \times 10^{5}}$
$=2.829 \mathrm{~ms}^{-2}$
(c)


Distance from centre
(d) $\Delta E p=m g \times h$
$=1.0 \times 10^{23} \times 2.829 \times 5.0 \times 10^{5}$
$=1.625 \times 10^{29} \mathrm{~J}$

Or
$\Delta E p=\frac{G m_{1} m_{2}}{\frac{1}{R i}-\frac{1}{R f}} \quad R i=3.39 \times 10^{6} \quad R f=(3.39+.5) \times 10^{6}$
$=1.625 \times 10^{29}$ Joules

