- **Circular Motion**

Inquiry question: Why do objects move in circles?

Uniform Circular Motion

Uniform circular motion occurs when an object travels in a circle at a constant speed.

- The **period** (T) is the time it takes for the object to make one complete revolution.
- The **distance** (d) completed during one revolution is the circumference of the circle ($2\pi r$).

The **average speed** of the object is given by: $\frac{2\pi r}{t}$

The object in circular motion is travelling at a constant speed but its changing direction constantly, therefore the velocity of the object is also changing. The magnitude of the velocity remains constant.

Centripetal acceleration

Since the object's velocity is constantly changing, the object must be accelerating. The direction of this acceleration is toward the centre of the circle and it is constant. This acceleration is called **centripetal acceleration.**

The centripetal acceleration of the object is given by: $a_c = \frac{v^2}{r}$

where:

a_c = the centripetal acceleration in $\frac{m}{s^2}$

v = the velocity in $\frac{m}{s}$

r = the radius of the circle in m

Centripetal force

As the object in uniform circular motion is accelerating, it must have some net force acting on it. The direction of this force is also toward the centre of the circle and it is constant. This net force is known as the **centripetal force.**

The centripetal force acting on the object is derived from: $F_c = m(\frac{v^2}{r})$:

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where:

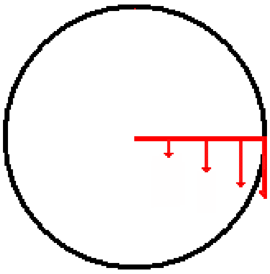
F_c = the centripetal force in N

v = the velocity in $\frac{m}{s}$

r = the radius of the circle in m

Average angular velocity

The diagram below illustrates angular velocity. While each point on the red arm travels at a different velocity, each point moves through the same angular displacement ($\Delta\theta$) in the same period of time.



This rate of change of angular displacement is the **average angular velocity** (ω)

$$\omega = \frac{\Delta\theta}{t}$$

where:
ω = angular velocity in $\frac{rad}{s}$ or $\frac{deg}{s}$

$\Delta\theta$ = the angular displacement in $radians$ (one revolution $= 360° = 2\pi\ radians$)

t = time in sec

***note:** in this equation, t is the time taken for the angular displacement and not the period, T.

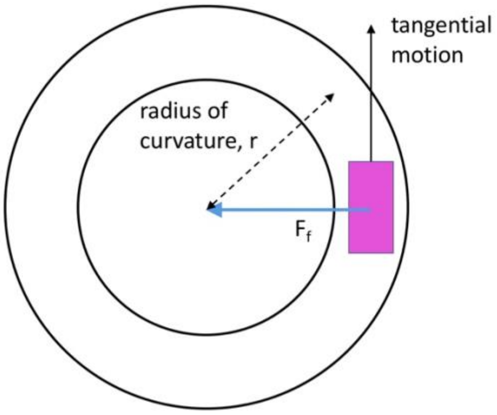
Forces and Circular Motion

In this section we will consider the forces acting on an object executing uniform circular motion by analysing the following three situations:

- car moving around horizontal circular bends
- a mass on a string
- objects on banked tracks

Car moving around horizontal circular bends

As a vehicle makes a circular turn, the inertia of the vehicle means that it wants to move along a straight line at a tangent to the curve. However, the vehicle will remain in uniform circular motion due to the friction between the tyres and the road. This frictional force is the centripetal force that allows the car to turn the corner: $F_f = F_c$

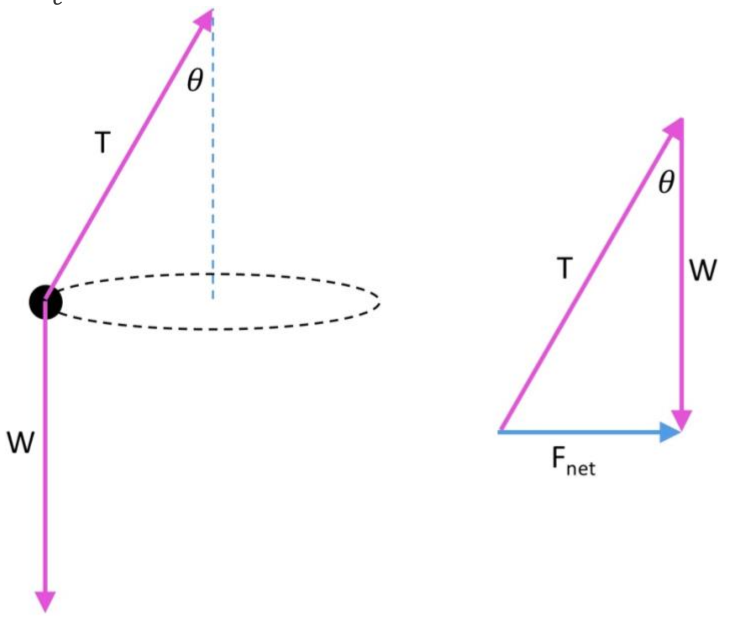


Questions may vary in the way the frictional force is presented. It may be a constant force or determined from the coefficient of friction, μ

- If the frictional force, F_f, is a constant, then: $F_c = F_f$
- If the frictional force is determined from the coefficient of friction, then: $F_c = F_f = \mu mg$

A mass on a string

Uniform circular motion can also occur when a mass on the end of a string is swung around in a circle at an angle from a fixed point. The string will have a tension force that allows this to occur. The object itself is undergoing uniform circular motion and so the net force on the object is equal to the centripetal force: $F_{net} = F_c$. The centripetal force is the resultant force of the weight force and the tension in the string: $F_{net} = F_c = W + T$.



The triangle that forms as a result of F_c, W and T can be used to analyse lengths/distances and forces by applying trigonometry.

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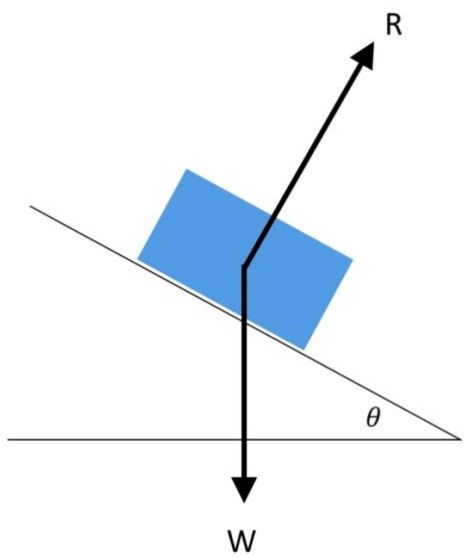
Objects on banked tracks

Banking a track or a road is achieved by tilting the surface up toward the centre of the circle. Velodromes and some roads are designed this way to allow for greater speeds and safety when turning. Banking allows some of the reaction force to contribute to the net force acting on a vehicle. This increases the magnitude of the net force and therefore, the magnitude of the centripetal force acting on the vehicle allowing increased speeds to be safely achieved.

***note:** friction plays a big role in any vehicle turning a corner and many problems of this nature will not consider the role of friction. Problems become more complex when friction is considered but can be broken down by analysing the vertical and horizontal components of friction.

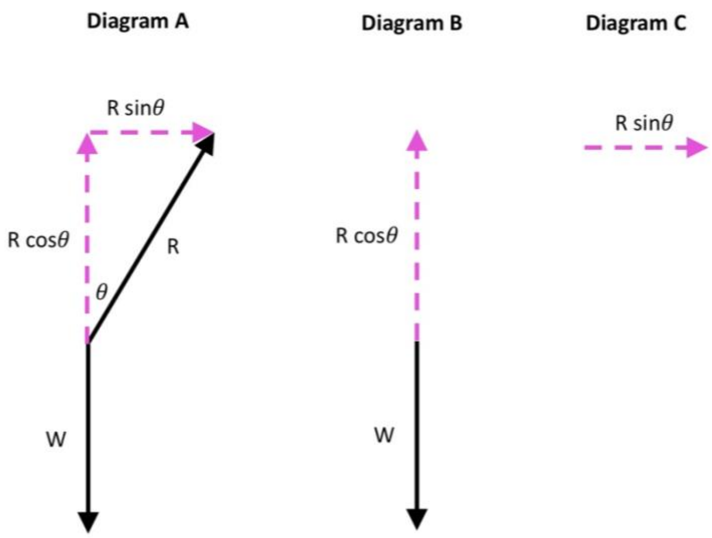
Banked tracks (ignoring friction):

The diagram below represents the forces acting on a vehicle in uniform circular motion on a banked track. The forces are the weight force, W, and the normal reaction force, R. The forces are unbalanced, and the resulting force is the centripetal force keeping the vehicle in uniform circular motion.



The diagrams break down the weight force and the normal reaction force into horizontal and vertical components:

- Diagram A shows all components
- Diagram B shows the vertical components.
- Diagram C shows the horizontal components



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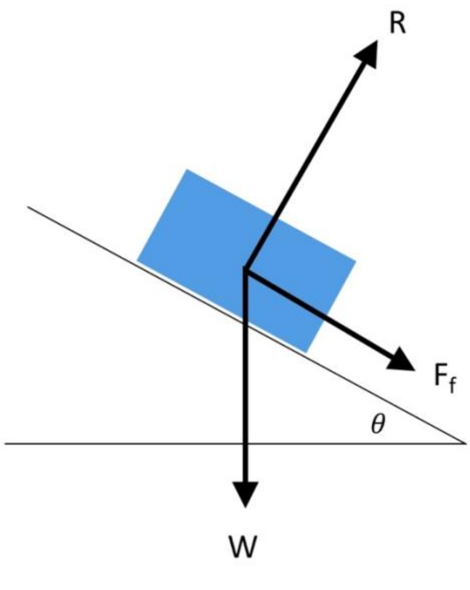
Analysing forces and the equations:

- Using diagram B: the vertical force component = 0. Therefore: $W = R\cos\theta$
- Using diagram C: the horizontal force component = $R\sin\theta$. This is the force that is responsible for the centripetal force, therefore: $F_c = R\sin\theta$

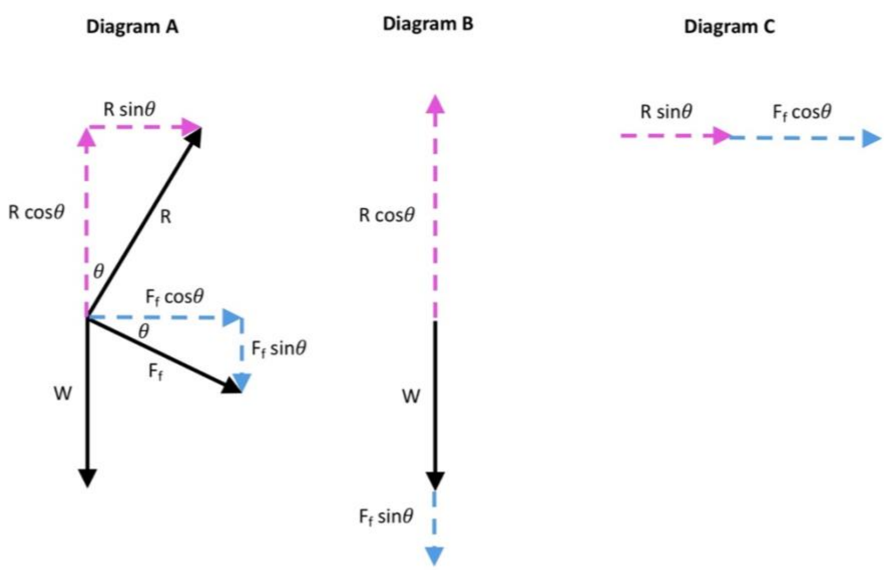
Banked tracks (considering friction):

The diagram below represents the forces acting on a vehicle in uniform circular motion on a banked track, including the friction force. The forces are the weight force, W, the normal reaction force, R and the friction force, F_f. The forces are unbalanced, and the resulting force is the centripetal force keeping the vehicle in uniform circular motion.



The diagrams break down the weight force, the normal reaction force and the friction force into horizontal and vertical components:

- Diagram A shows all components.
- Diagram B shows the vertical components.
- Diagram C shows the horizontal components



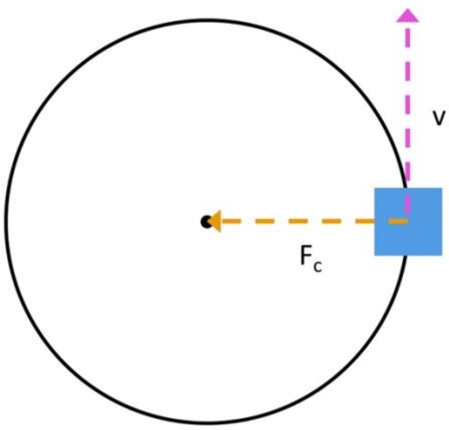
*note: the friction force contributes to the net force in both vertical and horizontal components.

Analysing forces and the equations:

- Using diagram B: the vertical force component = 0. Therefore: $W + F_f \, sin \, \theta = R \, cos \, \theta$
- Using diagram C: the horizontal force component = $R \sin \theta + F_f \cos \theta$. This is the force that is responsible for the centripetal force, therefore: $F_c = R \sin \theta + F_f \cos \theta$

Energy, Work and Circular Motion

Work is defined as the force (F) applied to move an object some displacement (s): $W = Fs$. As force and displacement are vectors, we correctly say that it is only the component of the force in the direction of motion that contributes to the work being done. This now results in the equation for work becoming: $W = Fs \cos \theta$



One of the features of circular motion, is that the centripetal force is always directed into the centre of the circle that is the path for the object. Even though the object is constantly changing direction, the motion at any instant is always tangential to the circle. This gives us the situation where the angle, θ, between the force and the direction of motion is 90°. If we apply this to our equation for work:

$W = Fs \cos \theta$

$W = Fs \cos 90°$

$W = 0$

Therefore, we conclude that no work is being done on an object in uniform circular motion.

Work is also defined as the change in kinetic energy. As the work done on an object in uniform circular motion is 0, there is no change in kinetic energy for the object in uniform circular motion. Kinetic energy is given by:

$KE = \frac{1}{2} m v^2$

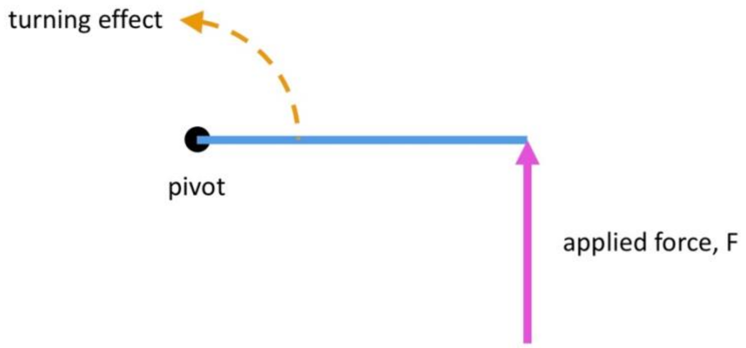
The kinetic energy depends only on the magnitude of the velocity and not on its direction. In uniform circular motion, only the direction of the velocity is changing, because the force is at right angles to the movement. Since the speed (i.e. the magnitude of the velocity) is constant, no work is being done and the energy remains constant.

For an object in uniform circular motion:

- No work is being done on the object
- There is no change in the object's kinetic energy
- The kinetic energy remains constant

Torque

Torque (τ) is the rotational effect that a force has on an object. The rotational effect occurs around some fixed point called a pivot. Torque is a vector quantity and the direction is described as being clockwise or anticlockwise.



The amount of torque that a force generates around the pivot point depends on several factors:

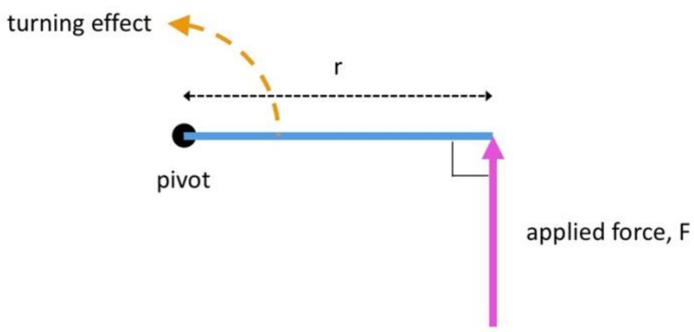
- magnitude of the applied force
- the distance from the pivot that the force is applied
- the direction that the force is applied

If the applied force is at right angles to the line between the pivot point and the force, torque, (τ), is given by the formula:

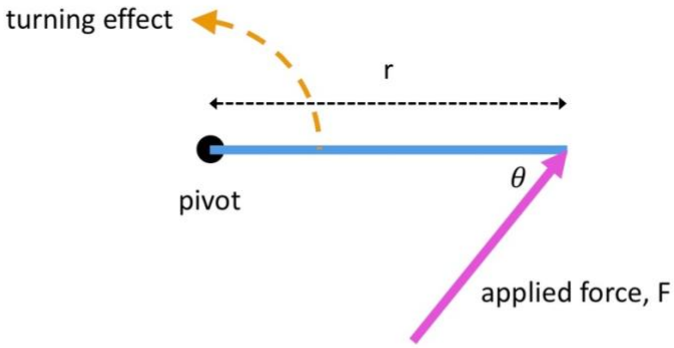
$$\tau = Fr$$

where:

τ is the torque in Nm
F is the applied force in N
r is the distance between the pivot point and the point where the force is applied in m



Often, the force is not applied at right angles and in these situations, it is only the component of the force that is perpendicular to the line between the pivot and the applied force that results in torque.



For these situations, we derive the following formula:

$\tau = F \perp r$

$\tau = Fr \sin \theta$